Name:

Class:

Date:

Year 12 Applications 2023
Test 1 Topic: Sequences

TIME ALLOCATION FOR THIS TEST: 55 minutes
WEIGHTING: 6% of year

Two sections

Syllabus points: 3.2.1 – 3.2.11

TOTAL

= ____ / 46

Material required/recommended for this test:

To be provided by the supervisor:
Question/answer booklets for Sections One.
SCSA 12 Applications Formula Sheet 2022

To be provided by the candidate:

Section 1:
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special materials: *drawing instruments, templates, no notes, formula sheet*

Section 2:
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special materials: *drawing instruments, templates, no notes, formula sheet, double sided sheet of A4 notes, up to 3 approved calculators*

Important note to candidates
No other items may be taken into the test room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the test room. If you have any unauthorised material with you, hand it to the teacher **before** reading any further.

Section	Working time	Marks	Score
1 - Resource free (non-calculator)	30	26	
2 – Resource rich (calculator)	25	20	

1. (7 marks-2,2,3)

Write down the first three terms for the following sequences:

a) $T_{n+1} = 4 - 2T_n$, $T_1 = 3$

$T_2 = 4 - 2(3) = -2$

$T_3 = 4 - 2(-2) = 8$

$3, -2, 8$ ✓ ✓

b) $T_{n+1} - T_n = 5$, $T_2 = -19$

$T_{n+1} = 5 + T_n$

$-24 \quad -19 \quad -14$ ✓ ✓

c) $4T_n = 16T_{n-1} - 10$, $T_4 = 11.5$

$T_n = 4T_{n-1} - 2\cdot5 \qquad T_4 = 11\cdot5$ ✓ For rewriting into T_n

$T_3 = \frac{11\cdot5 + 2\cdot5}{4} = 3\cdot5$

$T_2 = \frac{3\cdot5 + 2\cdot5}{4} = 1\cdot5$

✓ For T_3 and T_2

$T_1 = \frac{1\cdot5 + 2\cdot5}{4} = 1$

✓ For T_1

2. (3 marks-1,2)

The general term of a sequence is given by $T_n = 20 - 3n$

Calculate:

a) T_{56}

$T_{56} = 20 - 3(56)$

$= 20 - 168$

$= -148$ ✓

b) Show which term of the sequence is equal to -133.

$20 - 3n = -133$ ✓ Makes equation

$-20 \qquad -20$

$\frac{-3n}{-3} = \frac{-153}{-3}$

$n = 51 \quad \therefore T_{51} = -133$ ✓

3. (3 marks)

The 3[rd] term and 11[th] term of an arithmetic sequence are -6 and -38 respectively. Determine with reasons the 1[st] term of the sequence.

$\left.\begin{array}{l} a + 2d = -6 \\ a + 10d = -38 \end{array}\right\}$ ✓ $\quad \frac{2}{✓} \quad \frac{-2}{} \quad \frac{-6}{}$

$8d = -32$ ✓ Method to

$d = -4$ find $d = -4$

4. (2 marks)

The nth term of a sequence is defined as $T_n = 7 \times 6^{n-1}$

Write down the recursive rule of the sequence.

$T_{n+1} = 6T_n, \quad T_1 = 7$ ✓ ✓

5. 6 marks [2,2,2]

Consider the recurrence relation:

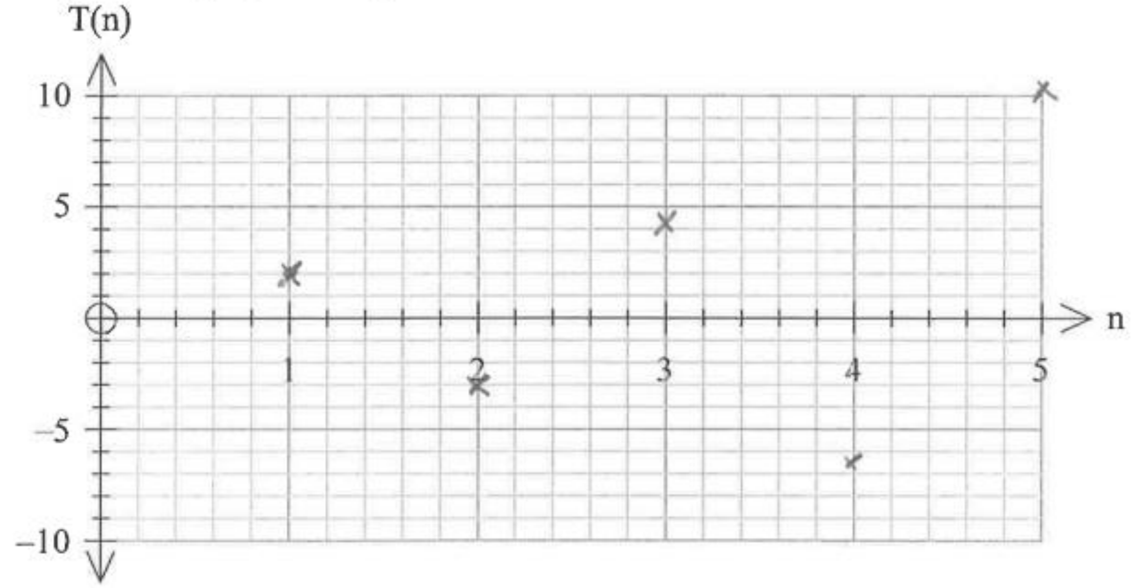
$$t_{n+1} = -\frac{3}{2}t_n, \ where\ t_1 = 2$$

a) Complete the table to values showing the term number (n) and the term value (t_n) for the first five terms of the sequence.

given

n	1	2	3	4	5
t_n	2	-3	$+9/2$ or $+4.5$	$-27/4$ or -6.75	$\frac{81}{8}$ or 10.125

b) Plot the graph on the given axes:



Does the recurrence relation have a long-term increasing, decreasing or steady-state solution? **Justify your answer**.

It switches between positive and negative terms that are diverging due to $r < -1$. It changes from long-term increasing to long-term decreasing, and back and forth.

Neither ✓
Explains ✓

6. **(2 marks)**

A recurrence relation is given as $\boldsymbol{A_{n+1} = (3-k)A_n + 2}$. Determine the value of k if $A_1 = 5$ *and* $A_2 = -12$. You may leave your answer as a fraction.

$-12 = (3-k)5 + 2$ ✓ substitutes correctly

$-12 = 15 - 5k + 2$

$-12 = 17 - 5k$

$-17 \quad -17$

$-29 = -5k$

$\frac{29}{5} = k$ ✓ value of k

7. **(3 marks)**

A first-order linear recurrence relation is in the form $T_{n+1} = bT_n + c, T_1 = a$

Calculate the values of b and c for the sequence below, showing your method.

$$-10, \quad -3, \quad \frac{1}{2}, \quad \ldots$$

$-10b + c = -3$

$-3b + c = \frac{1}{2}$ ✓

$\frac{-7b}{-7} = \frac{-3\frac{1}{2}}{-7}$

$b = \frac{1}{2}$ ✓

$c = -3\left(\frac{1}{2}\right) + \frac{1}{2}$

$c = -\frac{3}{2} + \frac{1}{2}$

$c = 2$ ✓

End of calculator-free section

Calculator Section **Name:** ______________________________

8. [4 marks 2,2]
The first four terms of a geometric sequence are:

$$90, \quad 60, \quad 40, \quad 26.\overline{6}, \quad \dots$$

a) Write a recurrence relation to define the geometric sequence.

$T_{n+1} = \frac{2}{3} T_n, \ T_1 = 90$

b) Write the $n^{th}\ term$ (general formula) to define the sequence.

$T_n = 90 \times 2/3^{(n-1)}$

or accept $135 \times 2/3^{n}$

9. [4 marks-2,2]
After the rollout of the Covid-19 vaccination in the UK, the r value dropped to 0.8. The number of new diagnoses each day followed a geometric sequence. Each 24-hour period results in a 20% drop in new diagnoses.

Covid-19 test results are released at 6pm each day. On 1st July 2021 at 6pm there were 33 000 new cases of Covid-19 diagnosed.

a) Calculate the number of cases diagnosed at 6pm on July 15th 2021.

$T_{15} = 1451.36$

$\therefore$ 1451 cases

deduct a mark for a decimal final answer

b) How many people were diagnosed (new cases) with Covid-19 between 6pm on 1st July and 6pm on 15th July inclusive?

Sum $T_1 \rightarrow T_{15} = 159194.57$

$\therefore$ 159194 cases (next person not diagnosed)

* only deduct one rounding mark in total for (a) and (b) combined

10. [8 marks – 2,2,2,2]

At the beginning of 1978, the population of Mandurah was approximately 10 000. Since then it has experienced an average growth rate of 6.5% per year, calculated at the end of the year

a) Create a recursive rule to model the population growth after n years.

$T_{n+1} = 1.065T_n, \ T_0 = 10\,000$ or $T_1 = 10650$ ✓ ✓

b) Complete the table below for the population of Mandurah, showing the population at the end of the year.

Year	1978	1979	1980	1981	1982
n	0	1	2	3	4
Population $P(n)$	10 000	10 650	11 342	12079 ✓	12864 ✓

deduct a mark for decimals

c) What would be the predicted population for Mandurah at the beginning of 2022?

$T_0 = 1978$) + 43

$T_{43} = 2021$ end of or start of 2022 ✓ correct reading of term (T_{43})

$= 149979.92\ldots$

$= 149980$ ✓

(Accept 149979 also for this Q)

* don't deduct a second rounding mark.

d) According to the *Local Government Act 1995,* a district can be classified as a city if its population exceeds 20 000. Using this definition, at the beginning of which year was the district of Mandurah first entitled to be classified as a city?

CAS spreadsheets $T_{11} = 19991.5$

$T_{12} = 21291.96$ ✓

$\therefore$ end of 1990 it was 1st a city

so start of 1991 ✓

11. [4 marks-1,1,2]

Apple trees are growing in an orchard. Over time, some of the trees stop producing enough apples and are removed at the end of the year in which this first occurs. Immediately afterwards, a fixed number of new apple trees will be planted.

The total number of apple trees growing in the orchard at the end of the nth year, $A(n)$, immediately after the planting of the new apple trees for that year, is modelled by the equation:

$$\boldsymbol{A(n+1) = 0.8A(n) + k \qquad A(1) = 18\,000}$$

a) What percentage of apple trees will be removed at the end of the year?

20%

b) Assume 100 new apple trees are planted at the end of each year. Determine how many apple trees will be growing in the orchard at the end of the third year, immediately after the planting of the new apple trees for that year.

$A_{n+1} = 0 \cdot 8 A_n + 100 \quad A_1 = 18000$

$T_3 = 11700$

c) Determine the number of new apple trees, k, that need to be planted at the end of each year so that there will always be 18 000 apple trees growing in the orchard.

$18000 = 18000(0 \cdot 8) + k$

$18000 = 14400 + k$

$3600 = k$

or 20% of $18000 = 3600$

End of assessment